

INTRODUCTION TO TRIGONOMETRY

The word Trigonometry is derived from three Greek words

TRI, GONO, METRON

Tri :- Three

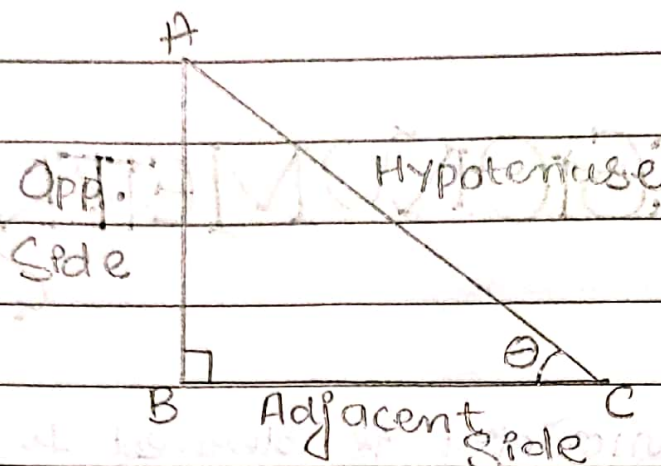
Gono :- Sides or Angles

Metron :- Measurement

Trigonometry is a branch of mathematics which deals with the study of three sides and three angles of a triangle.

Trigonometry is defined for a right triangle where hypotenuse is defined.

TRIGONOMETRIC RATIOS



Consider a right triangle ABC in which $\angle B = 90^\circ$ & $\angle C = \theta$.

∴ Trigonometric Ratios for $\triangle ABC$ are defined as follows :-

$$\sin \theta = \frac{\text{Opp. Side}}{\text{Hypotenuse}}$$

$$\operatorname{cosec} \theta = \frac{\text{Hypo}}{\text{Opp side}}$$

$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\sec \theta = \frac{\text{Hypo.}}{\text{Adj. side}}$$

$$\tan \theta = \frac{\text{Opp. side}}{\text{Adj. side}}$$

$$\cot \theta = \frac{\text{Adj. side}}{\text{Opp. side}}$$

RELATION BETWEEN TRIGONOMETRIC RATIOS

$$\textcircled{1} \operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sin \theta = \frac{1}{\operatorname{cosec} \theta}$$

$$\sin \theta \times \operatorname{cosec} \theta = 1$$

$$\textcircled{2} \sec \theta = \frac{1}{\cos \theta}$$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sec \theta \times \cos \theta = 1$$

$$\textcircled{3} \tan \theta = \frac{1}{\cot \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta \times \cot \theta = 1$$

$$4) \tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

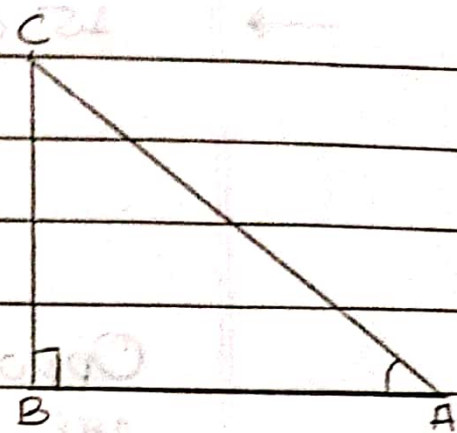
$$5) \tan \theta = \frac{\sec \theta}{\operatorname{cosec} \theta} \quad \cot \theta = \frac{\operatorname{cosec} \theta}{\sec \theta}$$

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

Opposite side = $3k = AB$

Hypotenuse = $4k = AC$

Adjacent side = ? = BC



By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$(4k)^2 = (3k)^2 + BC^2$$

$$16k^2 = 9k^2 + BC^2$$

$$BC^2 = 16k^2 - 9k^2$$

$$BC^2 = 7k^2$$

$$BC = \sqrt{7} k$$

$$\cos A = \frac{\sqrt{7} k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

* TRIGONOMETRIC RATIOS OF SOME
SPECIFIC ANGLES *

$\angle A^\circ$	0°	30°	45°	60°	90°
$\sin A$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos A$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan A$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not DEFINED
$\operatorname{cosec} A$	NOT DEFINED	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec A$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not Defined
$\cot A$	NOT DEFINED	$\sqrt{3}$	1	$1/\sqrt{3}$	0

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$\rightarrow (\sqrt{3}/2)(\sqrt{3}/2) + (1/2)(1/2)$
 $3/4 + 1/4$
 1

(ii) $2 \tan^2 45^\circ + \cos 30^\circ - \sin^2 60^\circ$

$\rightarrow 2 + 3/4 - 3/4$
 $2 + 0$
 2

$$\frac{\sin 30^\circ + \tan 45^\circ - \csc 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} + \frac{1}{2} + 1$$

$$\frac{3}{2} - \frac{2}{\sqrt{3}}$$

$$\frac{3}{2} + \frac{2}{\sqrt{3}}$$

$$\frac{3\sqrt{3} - 4}{2\sqrt{3}}$$

$$\frac{3\sqrt{3} + 4}{2\sqrt{3}}$$

$$\frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

$$\frac{(3\sqrt{3} - 4)^2}{27 - 16}$$

$\sin 2A = 2 \sin A$ is true when A

when $A = 0^\circ$

~~$$\sin 20^\circ = 2 \sin 0^\circ$$~~

~~$$2 \times 0 = 2 \times 0$$~~

~~$$0 = 0$$~~

$$0 \rightarrow (A) 0^\circ$$

If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 1/\sqrt{3}$,
 $0^\circ < A+B \leq 90^\circ$; $A > B$, find A and B .

Given,

$$\tan(A+B) = \sqrt{3}$$

$$\tan(A+B) = \tan 60^\circ$$

$$A+B = 60^\circ \quad \text{--- (1)}$$

$$\tan(A-B) = 1/\sqrt{3} \quad (\tan 30^\circ)$$

$$\tan(A-B) = \tan 30^\circ$$

$$A-B = 30^\circ \quad \text{--- (2)}$$

Subtract,

$$A + B = 60^\circ$$

$$A - B = 30^\circ$$

$$\hline 2A = 90^\circ$$

$$A = 45^\circ$$

$$A + B = 60^\circ$$

$$45^\circ + B = 60^\circ$$

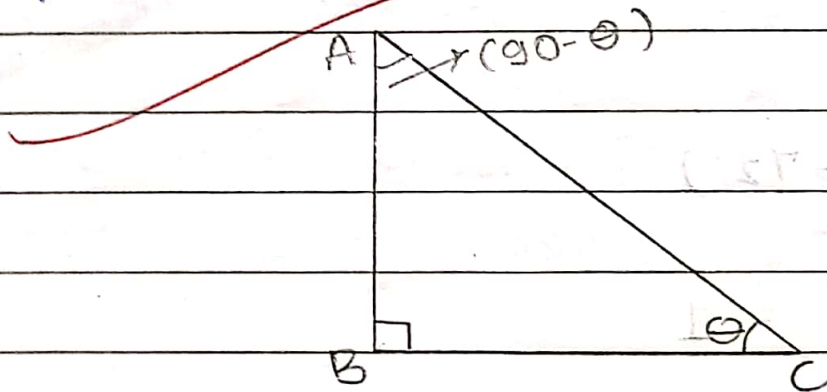
$$B = 15^\circ$$

$$\therefore A = 45^\circ$$

$$B = 15^\circ$$

TRIGONOMETRIC RATIOS OF COMPLEMENTARY * ANGLES *

Trigonometric ratios for complementary angles are defined as follows



$$\sin (90^\circ - \theta) = \cos \theta$$

$$\cos (90^\circ - \theta) = \sin \theta$$

$$\tan (90^\circ - \theta) = \cot \theta$$

$$\cot (90^\circ - \theta) = \tan \theta$$

$$\sec (90^\circ - \theta) = \operatorname{cosec} \theta$$

$$\operatorname{cosec} (90^\circ - \theta) = \sec \theta$$



$$\frac{\tan 26^\circ}{\cot 64^\circ}$$

$$\frac{\tan 26^\circ}{\tan (90^\circ - 64^\circ)}$$

$$\frac{\tan 26^\circ}{\tan 26^\circ}$$

$$\frac{\tan 26^\circ}{\tan 26^\circ}$$

$$1$$

$$\cos 48^\circ - \sin 42^\circ$$

$$\cos 48^\circ - \cos (90^\circ - 42^\circ)$$

$$\cos 48^\circ - \cos 48^\circ$$

$$0$$

$$\operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$\operatorname{cosec} 31^\circ - \operatorname{cosec} (90^\circ - 59^\circ)$$

$$\operatorname{cosec} 31^\circ - \operatorname{cosec} 31^\circ$$

$$0$$

✓ Show that :-

1) $\tan 48^\circ \cdot \tan 23^\circ \cdot \tan 42^\circ \cdot \tan 67^\circ = 1$

* LHS =

$$\tan 48^\circ \times \tan 42^\circ \times \tan 23^\circ \times \tan 67^\circ$$

$$\cot (90^\circ - 48^\circ) \times \tan 42^\circ \times \tan 23^\circ \times \cot (90^\circ - 42^\circ)$$

$$\cot 42^\circ \times \tan 42^\circ \times \tan 23^\circ \times \cot 23^\circ$$

$$\frac{1}{\tan 42^\circ} \times \tan 42^\circ \times \tan 23^\circ \times \frac{1}{\tan 23^\circ}$$

$$1$$

$$\therefore \text{LHS} = \text{RHS}$$

2) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

LHS =

$$\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$$

$$\sin (90^\circ - 38^\circ) \times \sin (90^\circ - 52^\circ) - \sin 38^\circ \sin 52^\circ$$

$$\sin 52^\circ \times \sin 38^\circ - \sin 38^\circ \times \sin 52^\circ$$

$$= 0$$

$$\therefore \text{LHS} = \text{RHS}$$

6] If A , B and C are interior angles of triangle ABC , then show that,

$$\sin \left(\frac{B+C}{2} \right) = \cos \frac{A}{2}$$

→ we have,

$$A + B + C = 180^\circ$$

$$B + C = 180^\circ - A$$

Divide by 2,

$$\frac{B+C}{2} = \frac{180^\circ - A}{2}$$

$$\frac{B+C}{2} = 90^\circ - \frac{A}{2}$$

$$\sin\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{A}{2}\right)$$

$$\sin\left(\frac{B+C}{2}\right) = \cos \frac{A}{2}$$

7] Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

$$\sin 67^\circ + \cos 75^\circ$$

$$\cos (90^\circ - 67^\circ) + \sin (90^\circ - 75^\circ)$$

$$\cos 23^\circ + \sin 15^\circ$$

TRIGONOMETRIC IDENTITIES

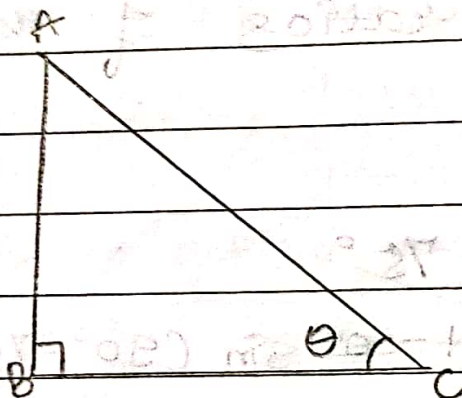
In $\triangle ABC$ if
 $\angle ACB = \theta$
 $\angle ABC = 90^\circ$

then,

i) $\sin^2 \theta + \cos^2 \theta = 1$

ii) $1 + \tan^2 \theta = \sec^2 \theta$

iii) $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$



$$\sin \theta = \frac{AB}{AC}$$

$$\operatorname{cosec} \theta$$

$$\cos \theta = \frac{BC}{AC}$$

$$\sec \theta$$

$$\tan \theta = \frac{AB}{BC}$$

$$\cot \theta$$

In $\triangle ABC$,
By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

Divide by AC^2

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

~~$$1 = \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$$~~

$$\cos^2 \theta + \sin^2 \theta = 1$$

Simplify
Divide by AB^2

~~$$\frac{AC^2}{AB^2} = \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2}$$~~

$$\sec^2 \theta = 1 + \tan^2 \theta$$

Divide by BC^2

$$\frac{AC^2}{BC^2} = \frac{AB^2}{BC^2} + \frac{BC^2}{\cancel{AB}BC^2}$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

2) Write all the other trigonometric ratios of $\angle A$ in terms of $\sec A$.

→ Using trigonometric identities,

$$\sin^2 A + \cos^2 A = 1$$

$$\sin^2 A + 1 = 1$$

$$\sec^2 A$$

$$\sin^2 A = 1 - \frac{1}{\sec^2 A}$$

$$\sin^2 A = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

also,

$$\cos A = \frac{1}{\sec A}$$

$$1 + \tan^2 A = \sec^2 A$$

$$\tan^2 A = \sec^2 A - 1$$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\frac{\cot A}{\tan A} = 1$$

$$\frac{\cot A}{\tan A}$$

$$\cot A = 1$$

$$\sqrt{\sec^2 A - 1}$$

$$\csc^2 A = 1$$

$$\sin A$$

$$\operatorname{cosec} A = \frac{1}{\sin A}$$

$$\frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$$

LHS =

$$\cos / 1 + \sin A + 1 + \sin A / \cos A$$

$$\frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A) \cos A}$$

$$\frac{\cancel{\cos^2 A} + 1 + 2 \sin A + \cancel{\sin^2 A}}{(1 + \sin A) \cos A}$$

$$\frac{1 + 1 + 2 \sin A}{(1 + \sin A) \cos A}$$

$$\frac{2 + 2 \sin A}{(1 + \sin A) \cos A}$$

$$\frac{2(1 + \sin A)}{(1 + \sin A) \cos A}$$

$$2 \sec A$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \sec \theta \operatorname{cosec} \theta$$

$$\frac{\tan \theta}{1 - \frac{1}{\tan \theta}} + \frac{1}{\frac{\tan \theta}{1 - \tan \theta}}$$

$$\frac{\tan \theta}{\tan \theta - 1} + \frac{1}{\tan \theta (1 - \tan \theta)}$$

$$\frac{\tan^3 \theta}{(\tan \theta - 1)} - \frac{1}{\tan \theta (\tan \theta - 1)}$$

$$\frac{\tan^3 \theta}{\tan \theta (\tan \theta - 1)} - \frac{1}{\tan \theta (\tan \theta - 1)}$$

$$\frac{\tan^3 \theta - 1}{\tan \theta (\tan \theta - 1)}$$

$$\frac{(\tan \theta - 1) (\tan^2 \theta + \tan \theta + 1)}{\tan \theta (\tan \theta - 1)}$$

$$\frac{\tan^2 \theta}{\tan \theta} + \frac{\tan \theta}{\tan \theta} + \frac{1}{\tan \theta}$$

$$1 + \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$1 + \frac{1}{\sin \theta \cos \theta}$$

$$1 + \sec \theta \operatorname{cosec} \theta$$

$$\frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

LHS =

$$\frac{1 + \sec A}{\sec A}$$

$$\frac{1}{\sec A} + \frac{\sec A}{\sec A}$$

$$1 + \cos A$$

$$\frac{1 + \cos A}{1} \times \frac{1 - \cos A}{1 - \cos A}$$

$$\frac{1 - \cos^2 A}{1 - \cos A}$$

$$\frac{\sin^2 A}{1 - \cos A}$$

~~So LHS = RHS~~

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

Divide by ~~$\sin A$~~ $\sin A$, LHS

$$\frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$\frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A - \operatorname{cosec} A + 1}$$

DATE: ___/___/___

$$\frac{\cot A + \operatorname{cosec} A - (\operatorname{cosec} A + \cot A) (\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$\frac{(\cot A + \operatorname{cosec} A) (1 - \operatorname{cosec} A + \cot A)}{\cot A - \operatorname{cosec} A + 1}$$

$$\operatorname{cosec} A + \cot A$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\text{C.P.N) } \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

→ LHS

$$\sqrt{\frac{1 \times \sin A \times (1 + \sin A)}{(1 - \sin A) (1 + \sin A)}}$$

$$\sqrt{\frac{(1 + \sin A)^2}{1^2 - \sin^2 A}}$$

$$\sqrt{\frac{(1 + \sin A)^2}{\cos^2 A}}$$

$$\frac{1 + \sin A}{\cos A}$$

$$\frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$\sec A + \tan A$$

$$\therefore \text{LHS} = \text{RHS}$$

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$(\sin^2 A + 2 \sin A \operatorname{cosec} A + \operatorname{cosec}^2 A) +$$

$$(\cos^2 A + 2 \cos A \sec A + \sec^2 A)$$

$$(\sin^2 A + 2 \sin A \operatorname{cosec} A + \operatorname{cosec}^2 A) +$$

$$(\cos^2 A + 2 \cos A \sec A + \sec^2 A)$$

$$\sin^2 A + 2 \sin A \operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A +$$

$$2 \cos A \sec A + \sec^2 A$$

$$1 + 2 \sin A \times \frac{1}{\sin A} + 1 + \cot^2 A + 2 \cos A \times \frac{1}{\cos A}$$

$$+ 1 + \tan^2 A$$

$$7 + \tan^2 A + \cot^2 A$$

$$\therefore \text{LHS} = \text{RHS}$$